**Investigating interesting sequences**

**In-class investigation**

**Solutions and marking key**

**Question 1 (a)**

|  |  |
| --- | --- |
| Solution  with | |
| Marking key/mathematical behaviours | Marks |
| * Includes the first term * Calculates each of the following four terms * Expresses terms in surd form where necessary | 1  2  1 |

**Question 1 (b)**

|  |  |
| --- | --- |
| Solution  By Pythagoras theorem | |
| Marking key/mathematical behaviours | Marks |
| * Identifies the use of Pythagoras theorem * Provides a statement of the relationship of the sides | 1  1 |

**Question 1 (c)**

|  |  |
| --- | --- |
| Solution  Pythagoras theorem: PX = | |
| Marking key/mathematical behaviours | Marks |
| * Substitutes into Pythagoras theorem * Calculates the length of the hypotenuse | 1  1 |

**Question 1 (d)**

|  |  |
| --- | --- |
| Solution            Construct a line, of length one unit, at right angles to the hypotenuse at point R. Join the end of the line to the central point to form the new right triangle. | |
| Marking key/mathematical behaviours | Marks |
| * Constructs lines at right angles to the hypotenuse * Draws lines of equal length * Labels diagram * Clarifies process used through description/ complete labelling | 1  1  1  1 |

**Question 1 (e)**

|  |  |
| --- | --- |
| Solution  For an hypotenuse of 64 (642 = 4096) there will be 4095 triangles as the number of triangles is one less than the number of units for which the length of the hypotenuse is the square root. | |
| Marking key/mathematical behaviours | Marks |
| * Identifies relationship with square of 64 * Identifies number of triangles * Links number of triangles to square root of the hypotenuse * Determine number of triangles | 1  1  1  1 |

**Question 2 (a)**

|  |  |  |  |
| --- | --- | --- | --- |
| Solution | Marking key/mathematical behaviours | | Marks |
| (i)  (ii)  (iii) | * Calculates first term * Calculates second term * Calculates third term | | 1  2  2 |
|  | |  | |

**Question 2 (b)**

|  |  |  |
| --- | --- | --- |
| Solution | Marking key/mathematical behaviours | Marks |
| (i)    (ii) | * Copies pattern * Extends pattern   Identifies first term  States recursion | 1  1  1  1 |

**Question 2 (c)**

|  |  |  |
| --- | --- | --- |
| Solution | Marking key/mathematical behaviours | Marks |
| Terms are approaching 1.618  The fraction being added each time is decreasing in value | * Identifies oscillation * Identifies limiting number * Identifies decreasing amount to be added to each term | 1  1  1 |

**Question 2 (d)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * Identifies two solutions * Expresses solutions in irrational form * Determines correct solutions for *α* and *β* | 1  1  1 |

**Question 2 (e)(i)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * Substitutes into expression * Simplifies expression and concludes terms are equal | 1  1 |

**Question 2 (e)(ii)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * Substitutes into expression * Expands by multiplying * Simplifies expression and concludes terms are equal | 1  1  1 |

**(4 +4 + 4 + 2 = 14 marks)**

3

|  |  |
| --- | --- |
| Mark | Description |
| 4 | |  |  | | --- | --- | | Sum of squares of successive terms in the Fibonacci sequence | Result | |  | 1 | |  | 2 | |  | 6 | |  | 15 | |  | 40 | |  | 104 |     (✓✓✓✓ One mark for each of four correctly calculated results. If, in addition, there are other results which are not correctly calculated, deduct 1 mark.) |

|  |  |
| --- | --- |
| Mark | Description |
| 4 | **(b) (i)**  ✓✓  **(ii)** ✓✓ |

|  |  |
| --- | --- |
| Mark | Description |
| 4 | |  |  | | --- | --- | | Sum of squares of consecutive pairs of terms in the Fibonacci sequence | Result | |  | 2 | |  | 5 | |  | 13 | |  | 34 | |  | 89 | |  | 233 | |  | 610 |     (✓✓✓✓ One mark for each of four correctly calculated results. If, in addition, there are other results which are not correctly calculated, deduct 1 mark.) |

|  |  |
| --- | --- |
| Mark | Description |
| 2 | **(d)** ✓✓ |